| Student Number: |
|---|
| Teacher: |

St George Girls High School

# Mathematics Advanced

## 2022 Trial HSC Examination

**General Instructions**

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- Calculators approved by NESA may be used
- A reference sheet is provided
- For questions in **Section I**, use the Multiple-Choice answer sheet provided
  For questions in **Section II:**
  - Answer the questions in the booklets provided
  - Show relevant mathematical reasoning and/or calculations
  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

**Total marks: 100**

**Section I – 10 marks** (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

**Section II – 90 marks** (pages 11 – 38)

- Attempt Questions 11 – 29
- Allow about 2 hours and 45 minutes for this section

| | |
|---|---|
| Q1 – Q10 | /10 |
| Q11 – Q13 | /12 |
| Q14 – Q16 | /12 |
| Q17 – Q19 | /13 |
| Q20 – Q21 | /14 |
| Q22 – Q24 | /13 |
| Q25 – Q26 | /12 |
| Q27 – Q29 | /14 |
| **Total** | **/100** |
| | % |

## Section I

**10 marks**
**Attempt Questions 1 – 10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. A jar contains 30 seashells. 19 of these are white and 11 are black.

   Leslie is to select two shells from the jar at random.

   What is the probability that Leslie selects two white shells?

   (A) $\frac{19}{30} \times \frac{18}{29}$

   (B) $\frac{19}{30} + \frac{18}{30}$

   (C) $\frac{19}{30} + \frac{18}{29}$

   (D) $\frac{19}{30} \times \frac{18}{30}$

2. A circle has the equation $(x+2)^2 + (y+3)^2 = r^2$.

   What is the value of $r^2$ such that the $x$-axis is a tangent to the circle?

   (A) 2

   (B) 3

   (C) 4

   (D) 9

3. What is the value of $E(X)$ for the probability distribution table below?

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.45 | $k$ | 0.20 |

(A) 0.45

(B) 1.00

(C) 1.75

(D) 2.75

4. For a particular function $y = f(x), f'(e) = 0$ and $f''(e) = \pi$.

How could the point where $x = e$ be described?

(A) A maximum turning point.

(B) A minimum turning point.

(C) A point of inflection.

(D) A horizontal point of inflection.

5. A computer technician notes that 40% of computers fail because of their hard drive, 25% because of the monitor, 20% because of a disk drive, and 15% because of the microprocessor.

If the problem is not in the monitor, what is the probability that it is in the hard drive?

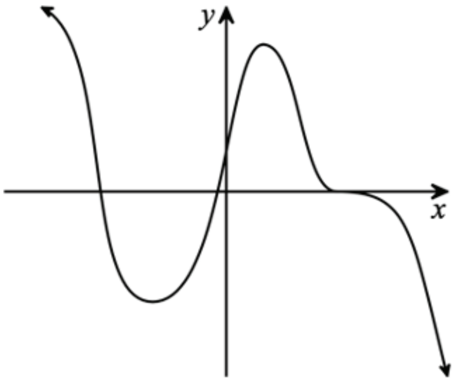
(A) 0.333

(B) 0.400

(C) 0.417

(D) 0.533

6. The graph of $y = f'(x)$ is shown below.




How many points of inflection does $y = f(x)$ have?

(A) 0

(B) 1

(C) 2

(D) 3

7. A series is given as $1 - 3k + 9k^2 + \dots$ . What is the value of $k$ if the sum to infinity is $\frac{3}{4}$?

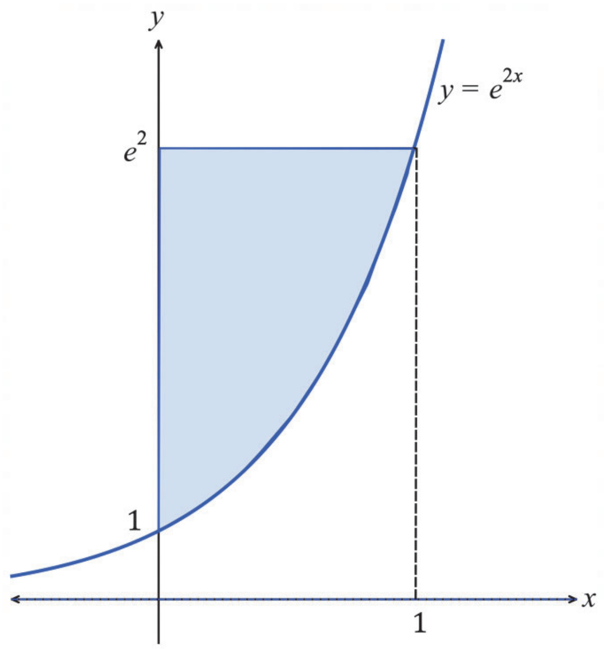
(A) $k = -\dfrac{7}{9}$

(B) $k = -\dfrac{1}{9}$

(C) $k = \dfrac{1}{9}$

(D) $k = \dfrac{7}{9}$

8.




Which of the following gives the correct expression to calculate the area of the shaded region above?

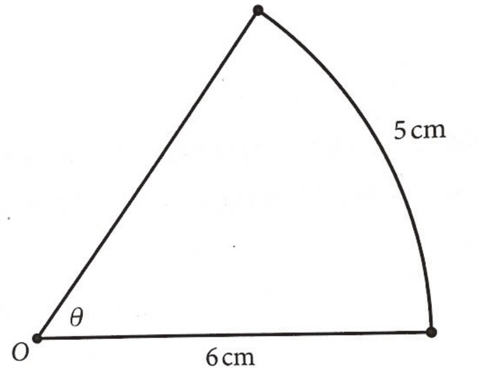
(A) $\displaystyle\int_0^1 e^{2x}\,dx$

(B) $\displaystyle\int_0^1 \frac{1}{2}\ln x\,dx$

(C) $\displaystyle\int_1^{e^2} e^{2y}\,dy$

(D) $\displaystyle\int_1^{e^2} \frac{1}{2}\ln y\,dy$

9. What is the area of the sector below, centred at $O$?




(A) $1.88\ \text{cm}^2$

(B) $15\ \text{cm}^2$

(C) $21.6\ \text{cm}^2$

(D) $10.42\ \text{cm}^2$

10. It is known that $f(x)$ is an odd function and $g(x)$ is an even function.

Given that $f(2) = 2$ and $g(2) = -2$, what is the value of $f\big(g(-2)\big) + g\big(f(-2)\big)$?

(A) $-4$

(B) $-2$

(C) $0$

(D) $4$

**Question 11** (2 marks)

Solve $|2x - 3| = 4$. **2**

**Question 12** (3 marks)

Find:

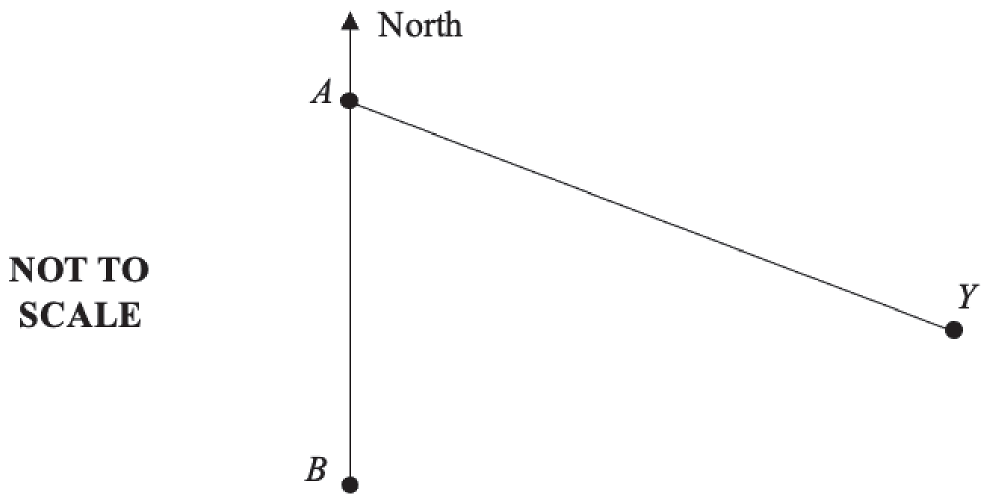
(a) $\frac{d}{dx}\left(x\sqrt{x}\right)$. **1**

(b) $\frac{d}{dx}\left(\ln\left(\frac{1-3x}{1+2x}\right)\right)$. **2**

**Question 13** (7 marks)

A yacht leaves Port A on a bearing of 120° and sails for three hours at an average speed of 15km/h to its destination where it stops.

At the same time, a speed boat also leaves from Port A and travels due south to Island B that is 30km from the port.




(a) Calculate the distance of the yacht from Island B to the nearest kilometre. **2**

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(b) Find the bearing of Island B from the yacht to the nearest degree. **3**

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(c) After spending several hours at the island, the speed boat travels due north back to Port A. **2**

How far south of the port will the speed boat be when it is directly west of the yacht?

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**Question 14** (3 marks)

Find:

(a) $\int (2 + 5x^2)\, dx$. **1**

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(b) $\int 5 \sin x \cos^3 x\, dx$. **2**

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**Question 15** (3 marks)

Find the exact gradient of the tangent to the curve $y = x \tan x$ at the point where $x = \frac{\pi}{6}$. **3**

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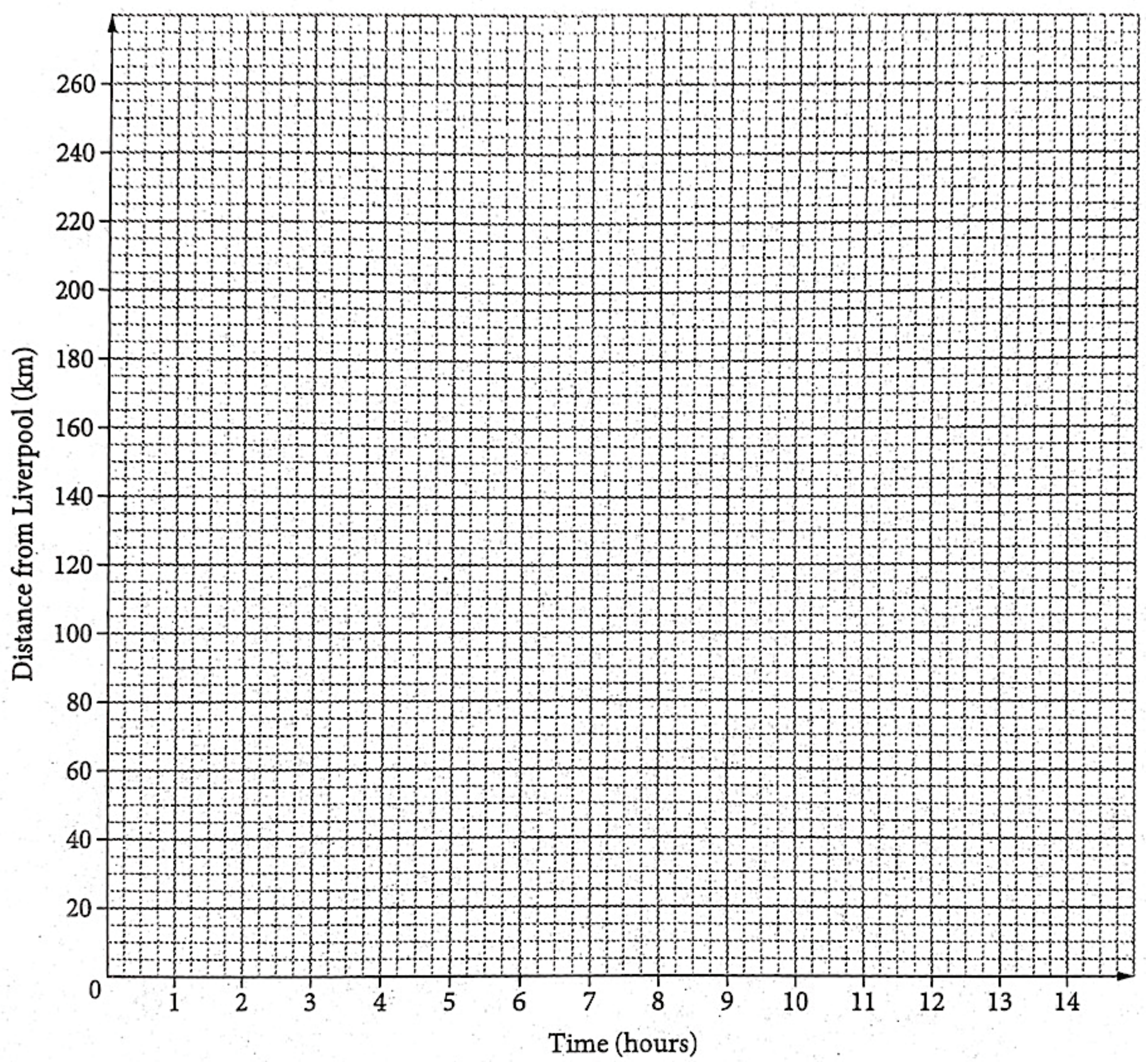
**Question 16** (6 marks)

Chris and Ann are participating in a charity bicycle ride between Canberra and Liverpool.

Chris leaves Canberra and rides to Liverpool, a distance of 250 km, at an average speed of 20km/h. His distance from Liverpool is modelled by the equation $C = 250 - 20t$, where $C$ is his distance from Liverpool and $t$ is the time in hours he has been riding.

(a) On the grid below, sketch the graph of this model and label it 'Chris'. **1**



(b) Ann rides in the opposite direction and leaves from Berrima, a town located 90km from Liverpool. She begins riding at the same time as Chris and rides at an average speed of 12km/h towards Canberra. **2**

By drawing a line on the grid above, or otherwise, find the value of $t$ when Chris and Ann pass each other.

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(c) Chris and Ann are initially 160km apart. Using the graphs drawn, or otherwise, find the value of $t$ when Chris and Ann are next 160km apart. **1**

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(d) Find the value of $t$ when the riders have ridden a total of 264km. **2**

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**Question 17** (3 marks)

Without using calculus, sketch the graph of $y = 2 - \frac{1}{x+4}$, showing any intercepts and asymptotes. **3**

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**Question 18** (6 marks)

A runner is training for a long-distance event.

The first week she runs 1.2 km.

The second week she runs 1.8 km.

The third week she runs 2.7 km, and so on, adding on half of the previous week.

(a) How far does she run in the fourth week? **1**

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(b) How far does she run in **total** after the first 6 weeks? **2**

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(c) The event she is training for is 45 km. **3**

In which week will she first exceed this distance?

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**Question 19** (4 marks)

Ash and Naomi complete a series of games. The series finishes when one player has won two games.

In any game, the probability that Ash wins is $\frac{3}{5}$ and the probability that Naomi wins is $\frac{2}{5}$.

(a) Draw a probability tree showing the possible outcomes, in a series of three games. **2**

(b) What is the probability that Naomi wins the series? **1**

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(c) What is the probability that Naomi wins at least one game? **1**

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**Question 20** (11 marks)

Let $f(x) = (2 - x)(x + 2)^3$.

(a) Show that $f'(x) = 4(x + 2)^2(1 - x)$. **2**

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(b) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. You may use $f''(x) = -12x(x + 2)$. **3**

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(c) Find the coordinates of all points of inflection of $y = f(x)$. **2**

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(d) Sketch the graph of $y = f(x)$ below, showing all intercepts, stationary points, and points of inflection. **3**

(e) Does $y = f(x)$ have a global maximum or global minimum in its natural domain? If so, specify where. **1**

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**Question 21** (3 marks)

Find the exact value of $\displaystyle\int_3^4 \frac{x}{x^2 - 8}\,dx$ in simplest form. **3**

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**End of Question 21**

**Proceed to Booklet 2 for Questions 22-29**

**Extra writing space.**
**If you use this space, clearly indicate which question you are answering.**

# Mathematics Advanced

## Section II Answer Booklet 2

| Student Number: |
|---|
| Teacher: |

**Section II**

**Booklet 2 – Attempt Question 22 – 29 (39 marks)**

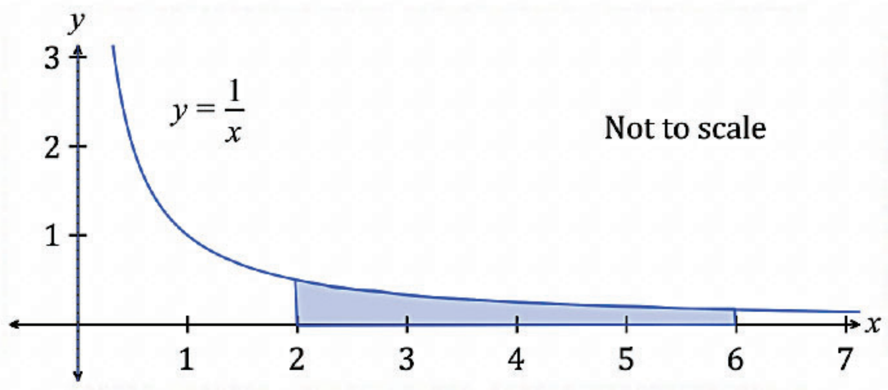
**Instructions**

- Write your Teacher's Name and Student Number at the top of this page.
- Answer the questions in the spaces provided. These spaces provide guidance for the expected length of response.
- Your responses should include relevant mathematical reasoning and/or calculations.
- Extra writing space is provided on pages 39—40 of Booklet 2. If you use this space, clearly indicate which question you are answering.

**Please turn over**

**Question 22** (4 marks)

Consider the curve $y = \frac{1}{x}$ sketched below.




(a) Find the area bounded by the curve, the $x$-axis, and the lines $x = 2$ and $x = 6$ using the Trapezoidal Rule with five function values. Give your answer correct to three decimal places. **2**

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(b) Calculate the same area by evaluating $\int_2^6 \frac{1}{x}\,dx$ . **1**
Give your answer correct to three decimal places.

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(c) Explain why there is a slight difference between your answers in part (a) and part (b). **1**

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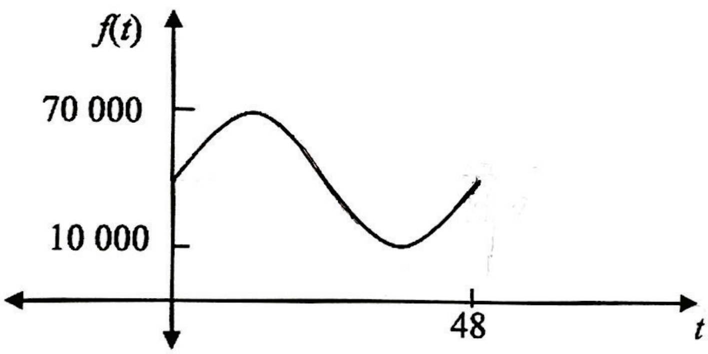
**Question 23** (2 marks)

The function $f(t) = a\sin\left(\frac{\pi}{24}t\right) + b$ is drawn below, where $0 \leq t \leq 48$.

The maximum value of $f(t)$ is 70 000 when $t = 12$. The minimum value of $f(t)$ is 10 000 when $t = 36$.




What are the values of $a$ and $b$? **2**

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**Question 24** (7 marks)

A particle starts to move from the origin along the $x$-axis.

Its velocity, measured in metres, at some time $t$ seconds, is given by $v = 4 - \frac{2}{3t+1}$ m/s.

(a) Explain why the particle is never at rest. **1**

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(b) Find the acceleration of the particle at $t = 3$ seconds. **2**

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(c) Find the exact distance travelled by the particle between $t = 0$ and $t = 5$ seconds. **3**

(d) Find the particle's limiting velocity. **1**

**Question 25** (4 marks)

At the beginning of the year 1935, 100 cane toads were introduced into Australia. Exactly 5 years later, the population had grown to 1000.

Assume that the number of cane toads is increasing exponentially and satisfies an equation of the form

$$N = N_0 e^{kt}$$

where $N_0$ and $k$ are constants and $t$ is measured in years from the start of 1935.

(a) Show that $N_0 = 100$ and $k = \frac{\ln 10}{5}$. **2**

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(b) How long does it take for the population to reach 2 million? Give your answer to the nearest year. **2**

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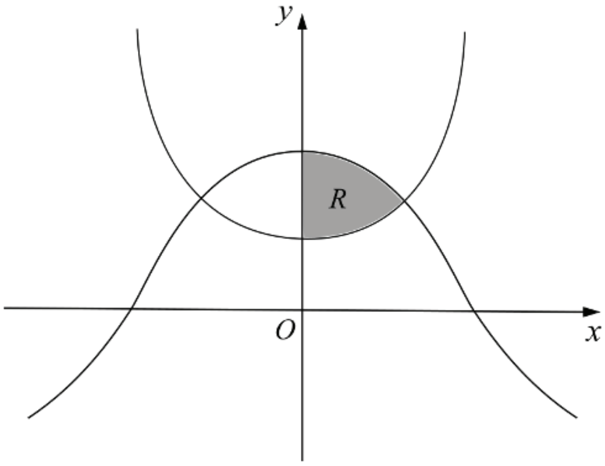
**Question 26** (8 marks)

(a) Show that $\frac{d}{dx}(\ln(\sec x + \tan x)) = \sec x$. **3**

The graph below shows the functions $y = \cos x$ and $y = \frac{1}{2}\sec x$.




(b) Show that the curves $y = \cos x$ and $y = \frac{1}{2}\sec x$ intersect in the first quadrant when $x = \frac{\pi}{4}$. **2**

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(c) Hence, find the exact area of the shaded region $R$. **3**

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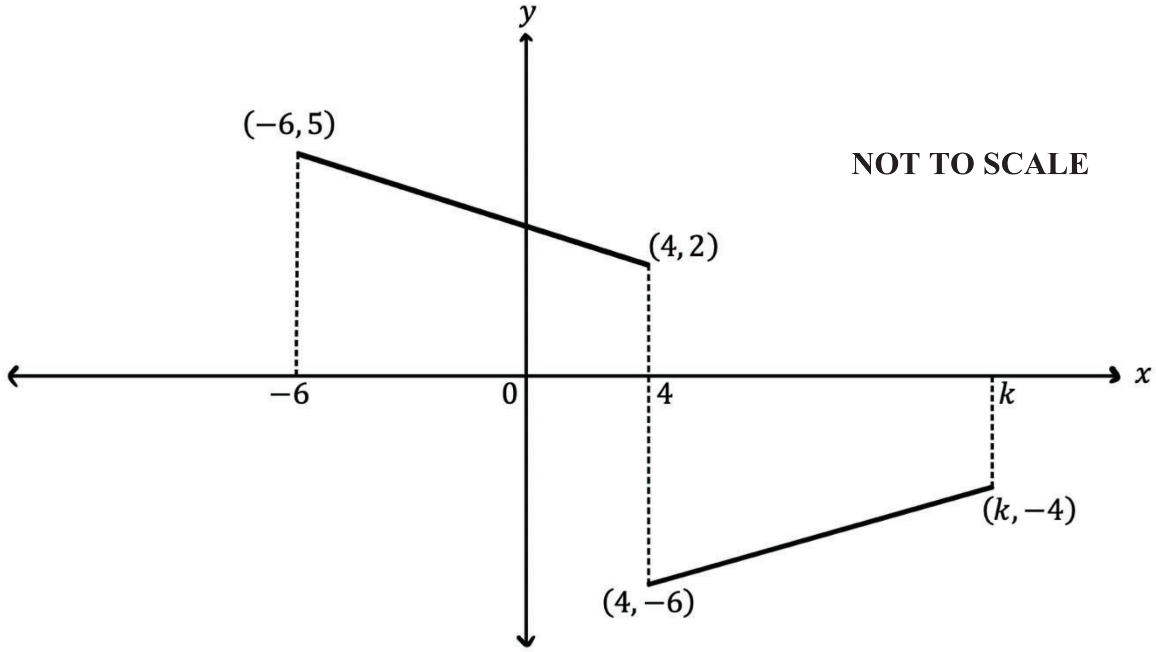
**Question 27** (3 marks)

Use the graph below to find the value of $k$ which satisfies $\int_{-6}^{k} f(x)\,dx = 0$. **3**

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**Question 28** (4 marks)

Point $A$ $(2e, 1)$ lies on the function $h(x)$. The tangent to $h(x)$ at $A$ has equation $y = \frac{x}{2e}$.

Point $B$ is the image of the point $A$ on the function $g(x) = 3h(2x + 4)$.

(a) Show that $B$ has coordinates $(e - 2, 3)$. **1**

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(b) Hence, find the equation of the tangent to $g(x)$ at point $B$, in general form. **3**

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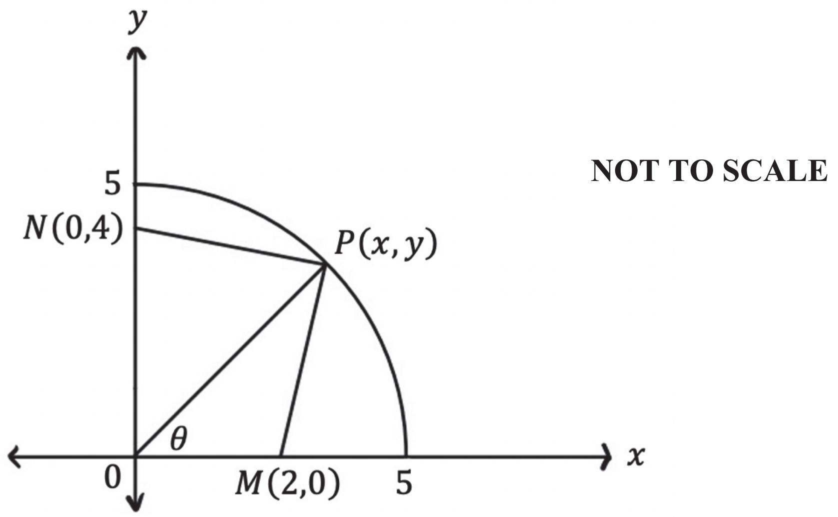
**Question 29** (7 marks)




The diagram above shows a part of the circle $x^2 + y^2 = 25$. The point $P(x, y)$ is on the circle, and point $O$ is the origin. Point $M$ has coordinates $(2, 0)$, point $N$ has coordinates $(0, 4)$, and $\angle MOP$ is measured in radians.

(a) Show that the area, $A$, of the quadrilateral $OMPN$ is given by **2**

$$A = 5\sin\theta + 10\cos\theta$$

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(b) Find the value of $\tan\theta$ which gives the maximum area $A$. **3**

(c) Hence find, in surd form, the coordinates of point $P$ when $A$ is maximum. **2**

**END OF PAPER**

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|---|---|
| Student Number: | |
| Teacher: | SOLUTIONS |

St George Girls High School

# Mathematics Advanced

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  - Marks may not be awarded for incomplete or poorly presented solutions, or where multiple solutions are provided

**Total marks: 100**

**Section I – 10 marks** (pages 3 – 7)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

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| | |
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| Q1 – Q10 | /10 |
| Q11 – Q13 | /12 |
| Q14 – Q16 | /12 |
| Q17 – Q19 | /13 |
| Q20 – Q21 | /14 |
| Q22 – Q24 | /13 |
| Q25 – Q26 | /12 |
| Q27 – Q29 | /14 |
| **Total** | **/100** |
| | % |

## Section I

**10 marks**
**Attempt Questions 1 – 10**
**Allow about 15 minutes for this section**

Use the multiple-choice answer sheet provided for Questions 1 to 10.

1. A jar contains 30 seashells. 19 of these are white and 11 are black.

   Leslie is to select two shells from the jar at random.

   What is the probability that Leslie selects two white shells?

   (A) $\frac{19}{30} \times \frac{18}{29}$

   (B) $\frac{19}{30} + \frac{18}{30}$

   (C) $\frac{19}{30} + \frac{18}{29}$

   (D) $\frac{19}{30} \times \frac{18}{30}$

2. A circle has the equation $(x+2)^2 + (y+3)^2 = r^2$.

   What is the value of $r^2$ such that the $x$-axis is a tangent to the circle?

   (A) 2

   (B) 3

   (C) 4

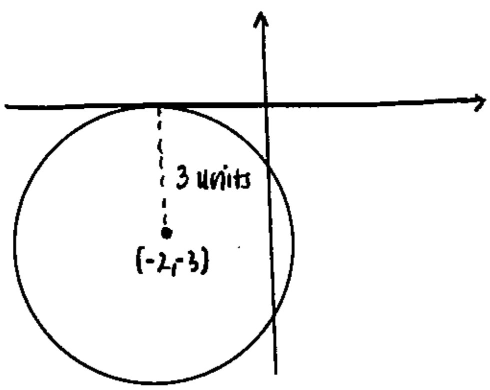
   (D) 9

centre : $(-2, -3)$

radius = 3 units

$\therefore r^2 = 3^2$

$= 9$



3. What is the value of E($X$) for the probability distribution table below?

| $x$ | 1 | 2 | 3 |
|---|---|---|---|
| $P(X = x)$ | 0.45 | $k$ 0.35 | 0.20 |

(A) 0.45

(B) 1.00

(C) 1.75

(D) 2.75

$k = 1 - 0.45 - 0.20$
$= 0.35$

$\therefore E(x) = 1 \times 0.45 + 2 \times 0.35 + 3 \times 0.2$
$= 0.45 + 0.7 + 0.6$
$= 1.75$

4. For a particular function $y = f(x), f'(e) = 0$ and $f''(e) = \pi$.

How could the point where $x = e$ be described?

(A) A maximum turning point.

(B) A minimum turning point.

(C) A point of inflection.

(D) A horizontal point of inflection.

$f'(x) = 0 \rightarrow$ stat. point
$f''(x) > 0 \rightarrow$ concave up

at $x = e$:
(min. tp.)

5. A computer technician notes that 40% of computers fail because of their hard drive, 25% because of the monitor, 20% because of a disk drive, and 15% because of the microprocessor.

If the problem is not in the monitor, what is the probability that it is in the hard drive?

(A) 0.333

(B) 0.400

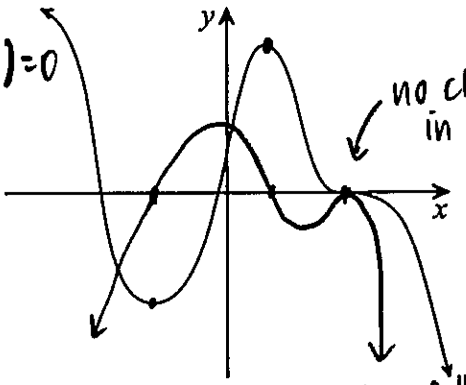
(C) 0.417

(D) 0.533

P(hard drive | not monitor) $= \dfrac{0.4}{0.4 + 0.2 + 0.15}$
$= \dfrac{0.4}{0.75}$
$= 0.5333\ldots$

6. The graph of $y = f'(x)$ is shown below.

Points of inflection: $f''(x) = 0$
+ change in concavity




no change in concavity

$y = f''(x)$

How many points of inflection does $y = f(x)$ have?

(A) 0

(B) 1

(C) 2

(D) 3

7. A series is given as $1 - 3k + 9k^2 + \dots$. What is the value of $k$ if the sum to infinity is $\frac{3}{4}$?

(A) $k = -\frac{7}{9}$

(B) $k = -\frac{1}{9}$

(C) $k = \frac{1}{9}$

(D) $k = \frac{7}{9}$

$\frac{-3k}{1} = -3k \quad \frac{\cancel{9}k^{\cancel{2}}}{-\cancel{3}\cancel{k}} = -3k \quad \therefore r = -3k$

$S = \frac{a}{1-r}$

$\frac{3}{4} = \frac{1}{1+3k}$

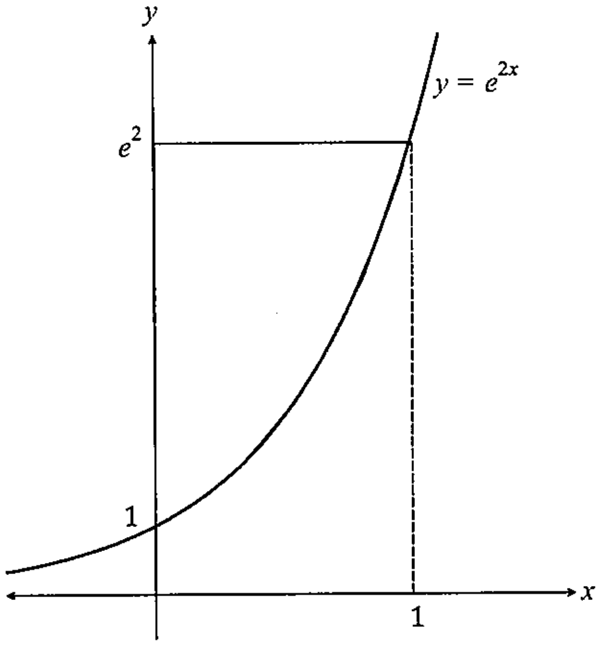
$3(1+3k) = 4$

$3 + 9k = 4$

$9k = 1$

$\therefore k = \frac{1}{9}$

8.




$y = e^{2x}$

$\ln y = \cancel{\ln} e^{2x}$

$\ln y = 2x$

$x = \frac{1}{2}\ln y$

$\therefore A = \int_1^{e^2} \frac{1}{2}\ln y \, dy$

Which of the following gives the correct expression to calculate the area of the shaded region above?

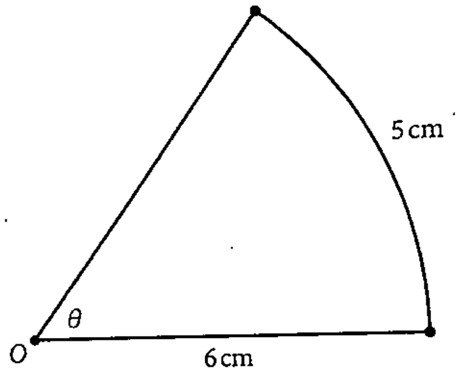
(A) $\int_0^1 e^{2x}\, dx$

(B) $\int_0^1 \frac{1}{2}\ln x\, dx$

(C) $\int_1^{e^2} e^{2y}\, dy$

(D) $\int_1^{e^2} \frac{1}{2}\ln y\, dy$

9. What is the area of the sector below, centred at $O$?




$l = r\theta$
$5 = 6\theta$
$\therefore \theta = \frac{5}{6}$

$A = \frac{1}{2}r^2\theta$
$= \frac{1}{2} \times \cancel{6^2} \times \frac{5}{\cancel{6}}$
$= \frac{1}{2} \times 30$
$= 15\ cm^2$

(A) $1.88\ cm^2$

(B) $15\ cm^2$

(C) $21.6\ cm^2$

(D) $10.42\ cm^2$

10. It is known that $f(x)$ is an odd function and $g(x)$ is an even function.

Given that $f(2) = 2$ and $g(2) = -2$, what is the value of $f(g(-2)) + g(f(-2))$?

(A) $-4$

(B) $-2$

(C) $0$

(D) $4$

$g(-2) = -2$ (even)
$f(-2) = -2$ (odd)

$f(g(-2)) + g(f(-2)) = f(-2) + g(-2)$
$= -2 + -2$
$= -4$

**Question 11** (2 marks)

Solve $|2x - 3| = 4$. **2**

$|2x - 3| = 4$

$2x - 3 = 4 \qquad -(2x - 3) = 4$

$2x = 7 \qquad 2x - 3 = -4$

$x = 7/2$ (1) $\qquad 2x = -1$

$x = -1/2$ (1)

$\therefore x = 7/2 , \ -1/2$

**Question 12** (3 marks)

Find:

(a) $\dfrac{d}{dx}(x\sqrt{x})$. **1**

$\dfrac{d}{dx}(x\sqrt{x})$

$= \dfrac{d}{dx}(x^{3/2})$

$\left.\begin{aligned} &= \frac{3}{2}x^{1/2} \\ &= \frac{3\sqrt{x}}{2} \end{aligned}\right\}$ (1)

(b) $\dfrac{d}{dx}\left(\ln\left(\dfrac{1 - 3x}{1 + 2x}\right)\right)$. **2**

$\dfrac{d}{dx}\left[\ln\left(\dfrac{1-3x}{1+2x}\right)\right]$

$= \dfrac{d}{dx}\left[\ln(1-3x) - \ln(1+2x)\right]$ (1)

$\left.\begin{aligned} &= \frac{-3}{1-3x} - \frac{2}{1+2x} \\ &= \frac{-3(1+2x) - 2(1-3x)}{(1-3x)(1+2x)} = \frac{-5}{(1-3x)(1+2x)} \end{aligned}\right\}$ (1)

# EXAMINER'S COMMENTS

Q12 METHOD 2 - Product Rule.

$$\frac{d}{dx}(x\sqrt{x}) \qquad u = x \quad v = x^{\frac{1}{2}}$$
$$u' = 1 \quad v' = \tfrac{1}{2}x^{-\frac{1}{2}}$$

$$\frac{d}{dx}(uv)$$
$$= vu' + uv'$$
$$= \sqrt{x} + \frac{x}{2\sqrt{x}}$$

Q12 Many students used the product rule rather than simplifying using their index laws. This waisted valuable time.

Q13 method 2 - Quotient Rule - Once again valuable time wasted using qotient rule rather than splitting to log up into a subroction.

$$p = \frac{u}{v}$$

$u = 1-3x$
$u' = -3$
$v = 1+2x$
$v' = 2$

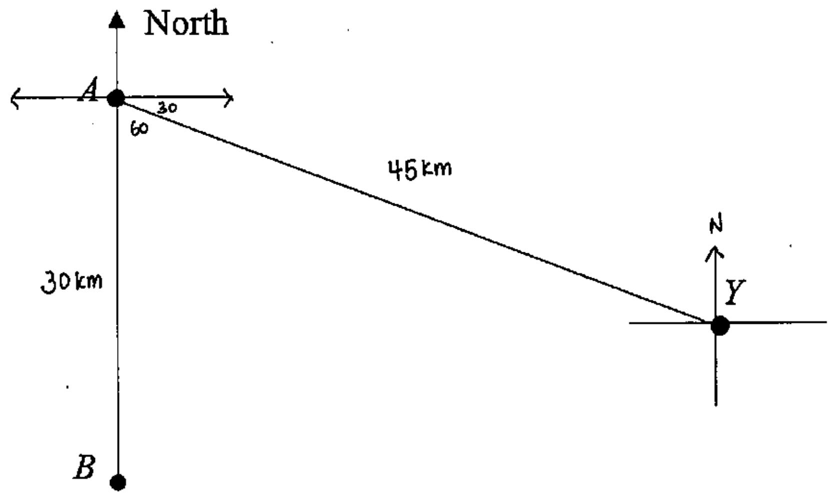
$$p' = \frac{vu' - uv'}{v^2}$$
$$= \frac{-3(1+2x) - 2(1-3x)}{(1+2x)^2}$$
$$= \frac{-3-6x-2+6x}{(1+2x)^2}$$
$$= \frac{-5}{(1+2x)^2} \quad (1)$$

$$\frac{d}{dx}\left[\ln f(x)\right]$$
$$= \frac{f'(x)}{f(x)}$$
$$= \frac{-5}{(1+2x)^2} \div \frac{1-3x}{1+2x}$$
$$= \frac{-5}{(1+2x)(1-3x)} \quad (1)$$

**Question 13** (7 marks)

A yacht leaves Port A on a bearing of 120° and sails for three hours at an average speed of 15km/h to its destination where it stops.

At the same time, a speed boat also leaves from Port A and travels due south to Island B that is 30km from the port.




(a) Calculate the distance of the yacht from Island B to the nearest kilometre. **2**

Sails for 3 hours at 15km/h

$\therefore 15 \times 3 = 45\text{km}$

$YB^2 = 30^2 + 45^2 - 2(30)(45)\cos 60°$ – 1

$YB^2 = 1575$

$YB = \sqrt{1575}$

$YB = 39.686269\ldots$

$YB \doteqdot 40\text{ km}$ – 1

# EXAMINER'S COMMENTS

13 c) 1 mark to establish a correct equation

1 mark to obtain correct answer

1 mark to obtain correct bearing

Q13 c) Other Methods.

$$\cos\angle AYB = \frac{45^2+40^2-30^2}{2(45)(40)} \leftarrow \text{USING ROUNDED OFF VALUE} \quad 1$$

~~$\cos\angle AYB = 0.7569\ldots$~~

$\angle AYB = 40.804437$

$\doteqdot 40°48'$ 1

~~Bearing $360-60-40°48'$~~
$= 259°T$ 1

$$\frac{\sin\angle AYB}{30} = \frac{\sin 60}{YB} \leftarrow \text{USING EXACT VALUE} \quad 1$$

$$\sin\angle AYB = \frac{30\sin 60}{YB}$$

$\sin\angle AYB = 0.65465\ldots$

$\angle AYB = 40°53'36.22'$ 1

$\doteqdot 41$

$\therefore$ Bearing $360-(60+41)$
$= 259°T$ 1

$$\cos\theta = \frac{30^2+(\sqrt{1575})^2-45^2}{2(30)(\sqrt{1575})} \quad 1$$

USING EXACT VALUE

$\theta = 79°$ 1

Bearing $= 180°+79°$
$= 259°T$ 1

OR using rounded off value

$$\cos\theta = \frac{30^2+40^2-45^2}{2(30)(40)} \quad 1$$

$\theta = 78°35'\ldots$ 1

Bearing
259° 1

OR USING ROUNDED VALUE

$$\frac{\sin\angle AYB}{30} = \frac{30\sin 60}{40} \quad 1$$

$\sin\angle AYB = 0.649519\ldots$

$\angle AYB = 40°30'19.26''$

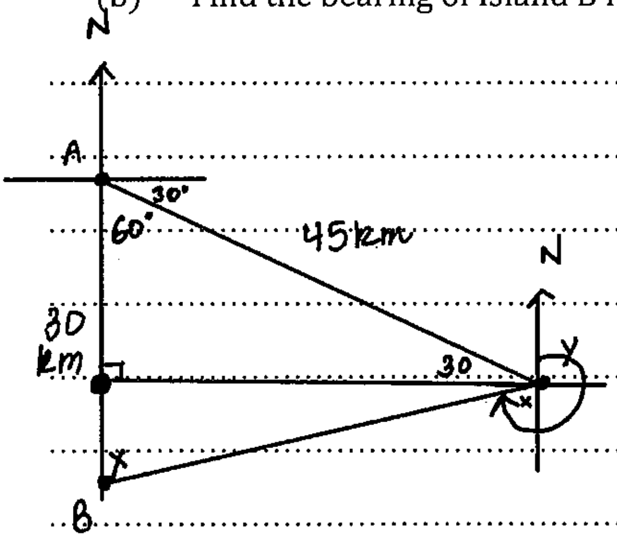
$\doteqdot 41°$ 1

$\therefore$ Bearing $= 360-(60+41)$
$= 259°T$ 1

c) Rounded answers accepted for (i) 40 (ii) 259 ⟷ Full marks if these rounded values were used ~~within c) resut.~~

If students rounded various answers in (b) and used these rounded values for (c), a rounding error was made (½ mark off)

(b) Find the bearing of Island B from the yacht to the nearest degree. **3**




$$\frac{\sin\angle YBA}{45} = \frac{\sin 60^\circ}{YB} \quad -1$$

$$= \frac{45\sin 60}{YB} \quad \text{OR} \quad = \frac{45\sin 60}{40} \leftarrow \text{using rounded value from (a)}$$

$\sin\angle YBA = 0.9819805061$ | $= 0.9742\cdots$

$\angle YBA = 79.106605\cdots$ | $= 76.9767\cdots^\circ$

$= 79^\circ 6' 23.78''$ | $= 76^\circ 58' 36.19''$

$\doteqdot 79^\circ$ | $-1 \quad \doteqdot 77^\circ$

$\therefore$ Bearing $180^\circ + 79^\circ$

$= 259^\circ T \quad -1$

Bearing $180^\circ + 77$

$= 257^\circ T$

(c) After spending several hours at the island, the speed boat travels due north back to Port A. **2**

How far south of the port will the speed boat be when it is directly west of the yacht?

$$\cos 60^\circ = \frac{x}{45} \quad -1$$

$$x = 45\cos 60^\circ$$

$$x = 22.5\text{km} \quad -1$$

OR

$$\sin 30^\circ = \frac{x}{45} \quad -1$$

$$x = 45\sin 30$$

$$x = 45 \times \tfrac{1}{2}$$

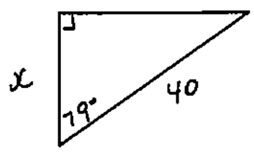
$$x = 22.5\text{ km} \quad -1$$

nearest degree part (ii)

$270 - 259 = 11^\circ$

$\sin 11 = \frac{x}{\sqrt{1575}}$

$x = 22.43$

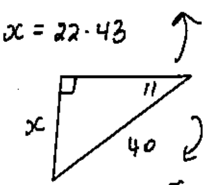



$\cos 79^\circ = \frac{x}{40}$

$x = 7.632\cdots$

$30 - 7.632\cdots$

$= 22.37$




$\sin 11 = \frac{x}{40}$

$x = 40\sin 11$

$= 7.63\cdots$

## EXAMINER'S COMMENTS

Q13 a) This question was done well by the majority of students.

Incorrect formula = no marks.

**Question 14** (3 marks)

Find:

(a) $\int (2 + 5x^2)\, dx$. 1

$= \underbrace{2x + \frac{5x^3}{3}}_{\frac{1}{2}\text{mk}} + \underbrace{C}_{\frac{1}{2}\text{mk}}$

(b) $\int 5 \sin x \cos^3 x\, dx$. 2

$= -5 \int -\sin x \cos^3 x\, dx$ — 1mk

$\left( \int f'(x)[f(x)]^n dx = \frac{1}{n+1}[f(x)]^{n+1} + C \quad n \neq -1 \right)$ — 1mk

Reference Sheet

$= -5 \times \frac{\cos^4 x}{4} + C$ — 1mk

or

Let $u = \cos x$

$\frac{du}{dx} = -\sin x$

$\sin x\, dx = -du$

$\int 5 \sin x \cos^3 x\, dx$

$= -5 \int u^3\, du$

$= \frac{-5u^4}{4} + C$

$= \frac{-5\cos^4 x}{4} + C$

**Question 15** (3 marks)

Find the exact gradient of the tangent to the curve $y = x \tan x$ at the point where $x = \frac{\pi}{6}$. 3

$y = x \tan x$

$y' = \tan x (1) + x \sec^2 x$ — 1mk

$= \tan x + x \sec^2 x$ — 1mk

When $x = \frac{\pi}{6}$

$y' = \tan \pi/6 + \pi/6 \times \sec^2\left(\frac{\pi}{6}\right)$

$= \frac{1}{\sqrt{3}} + \frac{\pi}{6} \times \left(\frac{1}{\cos^2(\pi/6)}\right)$

$= \frac{1}{\sqrt{3}} + \frac{\pi}{6} \times \left(\frac{2}{\sqrt{3}}\right)^2$

$= \frac{1}{\sqrt{3}} + \frac{\pi}{6} \times \frac{4}{3}$

$= \frac{1}{\sqrt{3}} + \frac{4\pi}{18}$

$= \frac{1}{\sqrt{3}} + \frac{2\pi}{9}$

$\therefore$ gradient of tangent is $\frac{1}{\sqrt{3}} + \frac{2\pi}{9}$

— 1mk for the answer.

**EXAMINER'S COMMENTS**

Question 14

a) Generally, done well. However, a few students forgot to include c and so lost ½ a mark.

b) Very poorly attempted. The formula for reverse chain rule is provided on the reference sheet and should have been used.
1 mark was deducted if students failed to show all the working, including $\int 5 \sin x \cos^3 x \, dx$
$= -5 \int -\sin x \cos^3 x \, dx$

Question 15

a) Students were able to apply the product rule formula quite well. However, a number of them made calculational errors such as did not know that $\cos(\pi/6) = \frac{\sqrt{3}}{2}$

Students are encouraged to leave their answer in the most simplified form. However, no marks were deducted if the final answer was not left in the most simplified form.

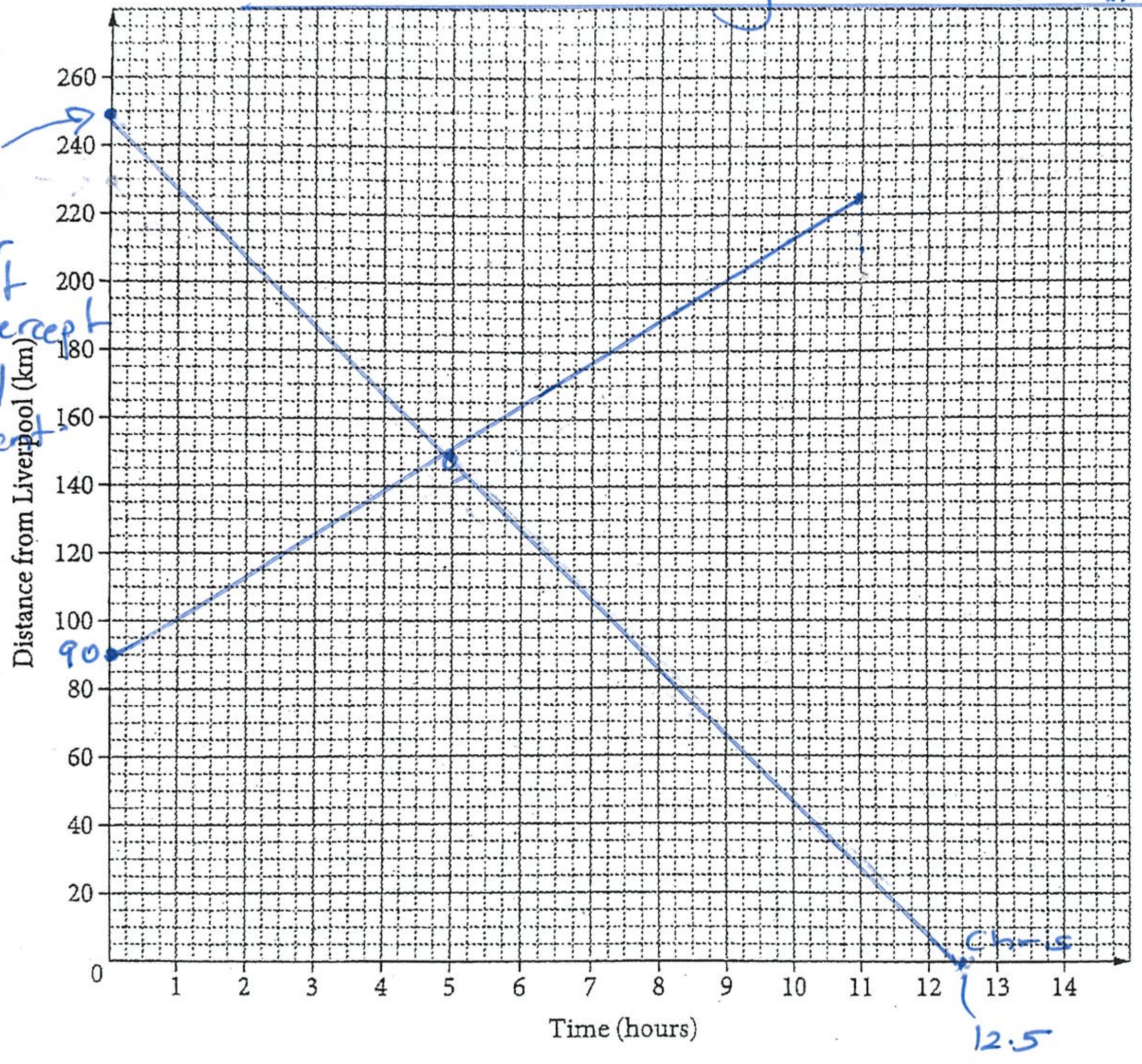
**Question 16** (6 marks)

Chris and Ann are participating in a charity bicycle ride between Canberra and Liverpool.

Chris leaves Canberra and rides to Liverpool, a distance of 250 km, at an average speed of 20km/h. His distance from Liverpool is modelled by the equation $C = 250 - 20t$, where $C$ is his distance from Liverpool and $t$ is the time in hours he has been riding.

(a) On the grid below, sketch the graph of this model and label it 'Chris'. **1**

Distance travelled by Chris and Ann.




½ mk – for correct y-intercept and gradient.

½ mk with correct x-intercept and gradient.

$C = 90 + 12t$

$= 90 + 12(11)$

$= 222$

(b) Ann rides in the opposite direction and leaves from Berrima, a town located 90km from Liverpool. She begins riding at the same time as Chris and rides at an average speed of 12km/h towards Canberra. **2**

By drawing a line on the grid above, or otherwise, find the value of $t$ when Chris and Ann pass each other.

Graphically

1mk – awarded for the correct graph – (no ½ mks)

1mk – awarded for $t=5$

$\therefore$ Chris and Ann pass each other after 5 hours

Algebraically

$C = 250 - 20t$

$A = 90 + 12t$

$90 + 12t = 250 - 20t$ – 1mk

$32t = 160$

$\therefore t = 5$ hours – 1mk

$\therefore$ Chris and Ann pass each other after 5h.

(c) Chris and Ann are initially 160km apart. Using the graphs drawn, or otherwise, find the value of $t$ when Chris and Ann are next 160km apart. **1**

Graphically

When $t = 10$ hours

(Chris = 50km

Ann = 210km)

Algebraically

$90 + 12t - (250 - 20t) = 160$

$90 + 12t - 250 + 20t = 160$

$32t - 160 = 160$

$32t = 320$

$\therefore t = 10$ h

No ½ mks

(d) Find the value of $t$ when the riders have ridden a total of 264km. **2**

Chris's distance = $20t$

Ann's distance = $12t$

$20t + 12t = 264$ — 1mk

$32t = 264$

$\therefore t = 8.25$ h — 1mk

(no error carried forward as complete lack of understanding of the question was displayed with other equations)

**EXAMINER'S COMMENTS**

16 b) Very poorly attempted. Students were not able to write down the equation for the distance travelled by Ann. They thought it was $90 - 12t$ instead of $90 + 12t$. Students are encouraged to read and understand the question before writing down their answers.

c) Again, poorly attempted due to lack of understanding of the question. No half marks were awarded for this question. Also, answers like $t = 0$ do not make sense as the question clearly says 'find the value of t when Chris and Ann are next 160 km apart.

d) Very poorly done. Again, lack of understanding of the question was displayed by the majority of students in the cohort.

**Question 17** (3 marks)

Without using calculus, sketch the graph of $y = 2 - \frac{1}{x+4}$, showing any intercepts and asymptotes. **3**

• asymptotes: $x = -4$
$y = 2$

• $x$-intercept: let $y = 0$

$2 - \frac{1}{x+4} = 0$

$\frac{1}{x+4} = 2$

$2(x+4) = 1$

$2x + 8 = 1$

$2x = -7$

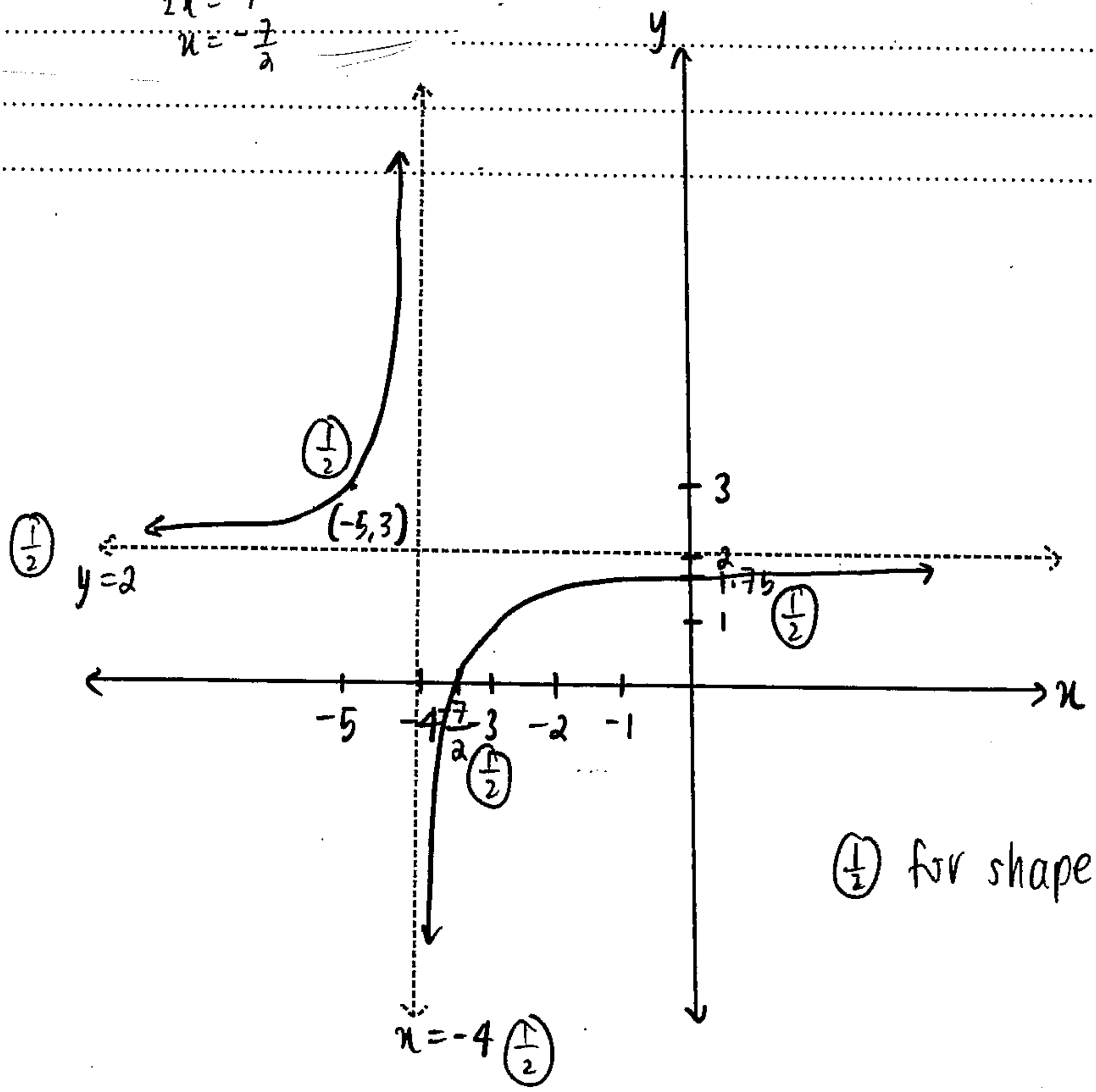
$x = -\frac{7}{2}$

• $y$-intercept: let $x = 0$

$y = 2 - \frac{1}{0+4}$

$= 2 - \frac{1}{4}$

$= \frac{7}{4}$




$\frac{1}{2}$ for shape

**MARKER'S COMMENTS - QUESTION** 17 Advanced trial. 2022

students need to.

(1) Show asymptotes • $y = 2$ $(\frac{1}{2})$

• $x = -4$ $(\frac{1}{2})$

(2) Show intercepts • x intercept $x = \frac{-7}{2}$ $(x = -3.5)$ $(\frac{1}{2})$

• y intercept $y = \frac{7}{4}$ $(y = 1\frac{3}{4})$. $(\frac{1}{2})$

(3) Demonstrate the correct shape for hyperbola. $(\frac{1}{2})$

(4) Show at least 1 point on each branch $(\frac{1}{2})$

. Many students did a good job on this question.

. A significant number of students failed to recognise this graph. as a hyperbola. Some only graphed 1 branch.

**Question 18** (6 marks)

A runner is training for a long-distance event.

The first week she runs 1.2 km. $1.2, 1.8, 2.7, \ldots$ $\frac{1.8}{1.2} = 1.5$

The second week she runs 1.8 km. $\frac{2.7}{1.8} = 1.5$ $\therefore r = 1.5$

The third week she runs 2.7 km, and so on, adding on half of the previous week.

(a) How far does she run in the fourth week? 1

$2.7 \times 1.5 = 4.05\text{km}$ OR: $a = 1.2,\ r = 1.5$: $T_n = ar^{n-1}$

$\therefore T_4 = 1.2 \times 1.5^{4-1}$

$= 1.2 \times 1.5^3$

$= 4.05\text{ km}$

(b) How far does she run in **total** after the first 6 weeks? 2

$S_n = \frac{a(r^n - 1)}{r-1}$

$\therefore S_6 = \frac{1.2(1.5^6 - 1)}{1.5 - 1}$ — 1

$= 24.9375\text{ km}$ — 1

OR: $1.2 + 1.8 + 2.7 + 4.05 + 6.075 + 9.1125 = 24.9375$

1 1

## MARKER'S COMMENTS - QUESTION 18

(a) . Most students successfully used $T_n = ar^{n-1}$ However, a significant number of students used A.P formula.

• Some students just wrote out each term by "common sense". these were generally successful

(b) $S_n = \frac{a(r^n - 1)}{r-1}$ Some incorrectly used A.P formula.

students that wrote out each term were often successful but careless mistakes do occur.

(c) The event she is training for is 45 km. **3**

In which week will she first exceed this distance?

Let $T_n = 45$.

$45 = 1.2 \times 1.5^{n-1}$ ① mark to here

$\frac{45}{1.2} = 1.5^{n-1}$

$37.5 = 1.5^{n-1}$ ①

$\ln 37.5 = \ln 1.5^{n-1}$

$\ln 37.5 = (n-1) \ln 1.5$

$n - 1 = \frac{\ln 37.5}{\ln 1.5}$

$n = \frac{\ln 37.5}{\ln 1.5} + 1$

$= 9.93872...$ $\frac{1}{2}$

$\doteqdot 10$ $\frac{1}{2}$

$\therefore$ She will exceed 45km in the 10th week.

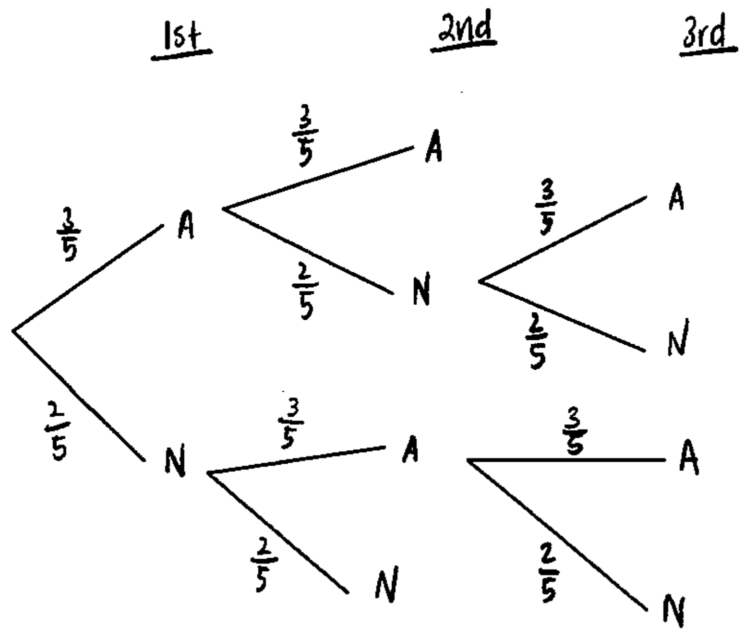
**Question 19** (4 marks)

Ash and Naomi complete a series of games. The series finishes when one player has won two games.

In any game, the probability that Ash wins is $\frac{3}{5}$ and the probability that Naomi wins is $\frac{2}{5}$.

(a) Draw a probability tree showing the possible outcomes, in a series of three games. **2**

Let A = Ash winning , N = Naomi winning




(b) What is the probability that Naomi wins the series? **1**

$P(\text{Naomi wins}) = P(A,N,N) + P(N,N) + P(N,A,N)$

$= \left(\frac{3}{5} \times \frac{2}{5} \times \frac{2}{5}\right) \times 2 + \left(\frac{2}{5} \times \frac{2}{5}\right)$

$= \frac{44}{125}$

(c) What is the probability that Naomi wins at least one game? **1**

$P(\text{at least } 1) = 1 - P(\text{none})$

$= 1 - P(A,A)$

$= 1 - \left(\frac{3}{5} \times \frac{3}{5}\right)$

$= \frac{16}{25}$

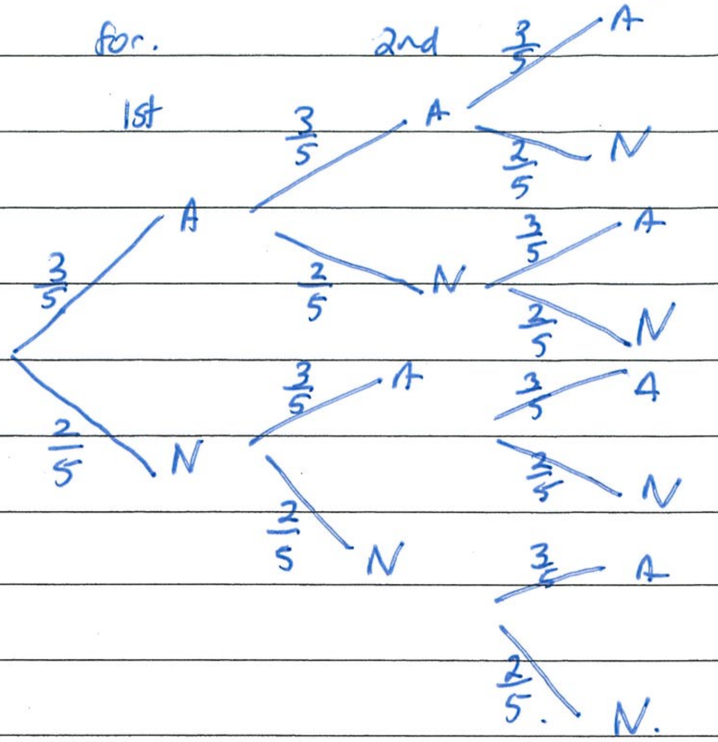
## MARKER'S COMMENTS - QUESTION 19

2(a) students received 1 mark 3rd.

for.

2nd

1st




if error carried forward-

(b) $\frac{36}{125}$.

(c) $\frac{98}{125}$.

**Question 20** (11 marks)

Let $f(x) = (2 - x)(x + 2)^3$.

(a) Show that $f'(x) = 4(x + 2)^2(1 - x)$. **2**

$f'(x) = (2-x) \times 3(x+2)^2 + (x+2)^3 \times -1$

$= (x+2)^2 [3(2-x) - (x+2)]$

$= (x+2)^2 [6 - 3x - x - 2]$

$= (x+2)^2 (4 - 4x)$

$= 4(x+2)^2(1-x)$ as required.

(b) Find the coordinates of the stationary points of $y = f(x)$ and determine their nature. You may use $f''(x) = -12x(x + 2)$. **3**

Stationary points occur when $f'(x) = 0$

ie $4(x+2)^2(1-x) = 0$

$\therefore x = -2$ & $x = 1$

$y = 0$ $y = 27$

$\therefore$ Stationary points are $(-2,0)$ & $(1,27)$.

$f''(-2) = -12(-2)(-2+2)$

$= 0$

$f''(1) = -12(1)(1+2)$

$= -12 \times 3$

$= -36 < 0$ $\cap$

$\therefore (1,27)$ is a maximum turning point.

Since $f'(-2) = f''(-2) = 0$ there is a possible horizontal point of inflection at $(-2,0)$.

| $x$ | $-3$ | $-2$ | $-1$ |
|---|---|---|---|
| $f''(x)$ | $-36$ | $0$ | $+12$ |
| concavity | $\frown$ | $\cdot$ | $\smile$ |

$\therefore$ Since there is a change in concavity, $(-2,0)$ is a horizontal point of inflection.

## EXAMINER'S COMMENTS

Q20 (a) • Use Reference sheet for correct product rule formula.

• It's a "show" question, so every line/step must be shown.

① mark for the very first line, showing correct use of product rule

① mark for all steps correct leading to the last line.

Q20 (b)

$\frac{1}{2}$ mark – for both correct $x$-values

$\frac{1}{2}$ mark – for both correct $y$-values

OR $\frac{1}{2}$ mark each for correct stationary points $(-2, 0)$ & $(1, 27)$.

For the point $(1, 27)$:

$\frac{1}{2}$ mark – for $f''(1) = -36 < 0$ testing concavity using 2nd derivative

$\frac{1}{2}$ mark – for then stating that $(1, 27)$ is a maximum turning

For the point $(-2, 0)$:

$\frac{1}{2}$-mark for table with correct values using the 2nd derivative.

$\frac{1}{2}$-mark for reason/statement, eg because the function has different concavities on either side of the point, then it is an actual point of inflection.

## EXAMINER'S COMMENTS

Students need to remember:

- The first derivative $f'(x)$ is the slope of the tangent line to the curve at the point $x$.
- The 2nd derivative $f''(x)$ tells us when the curve is concave up, concave down or there is a point of inflection.
- A point of inflection occurs at a point where $f''(x)=0$ AND the finction changes concavity. You have to make sure that the concavity actually changes at that point by using a table of values. From the table, you can conclude that because the finction has different concavities on either side of the point, then it is an actual point of inflection.

For Q20 (b):

Stating $f'(-2)=f''(-2)=0 \therefore (-2,0)$ is a horizontal point of inflection is not enough. You still need to test on either side of $x=-2$ and make sure there is a change in concavity also.

Some students used a table and tested the slope of the tangents which is also correct.

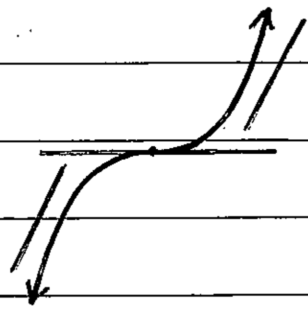
## EXAMINER'S COMMENTS

Using $f'(x) = 4(x+2)^2(1-x)$

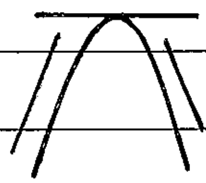
| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ | $2$ |
|---|---|---|---|---|---|---|
| $f'(x)$ | 16 | 0 | 8 | 16 | 0 | $-64$ |
| slope of tangent | / | — | / | / | — | \ |

$\left(\frac{1}{2}\right)$ $\left(\frac{1}{2}\right)$

From this table we can see:

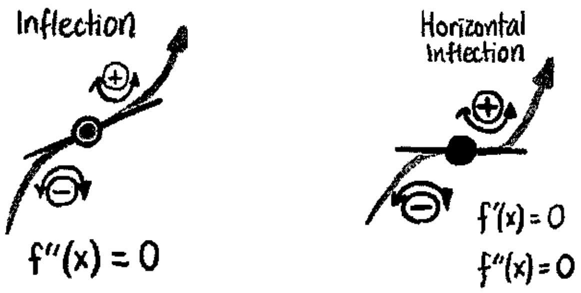


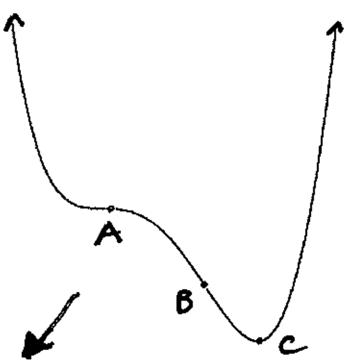
$\therefore (-2, 0)$ is a horizontal point of inflection. $\left(\frac{1}{2}\right)$



$\therefore (1, 27)$ is a maximum turning point. $\left(\frac{1}{2}\right)$

Remember there are 2 types of inflection points:




A is a stationary point of inflection known as a "horizontal point of inflection", B is a non-stationary point of inflection known as a "point of inflection", and C is a minimum turning point.

$f''(x) = -12x(x+2)$

(c) Find the coordinates of all points of inflection of $y = f(x)$. **2**

Points of inflection occur when $f''(x) = 0$

ie $-12x(x+2) = 0$

$x = 0$ & $x = -2$

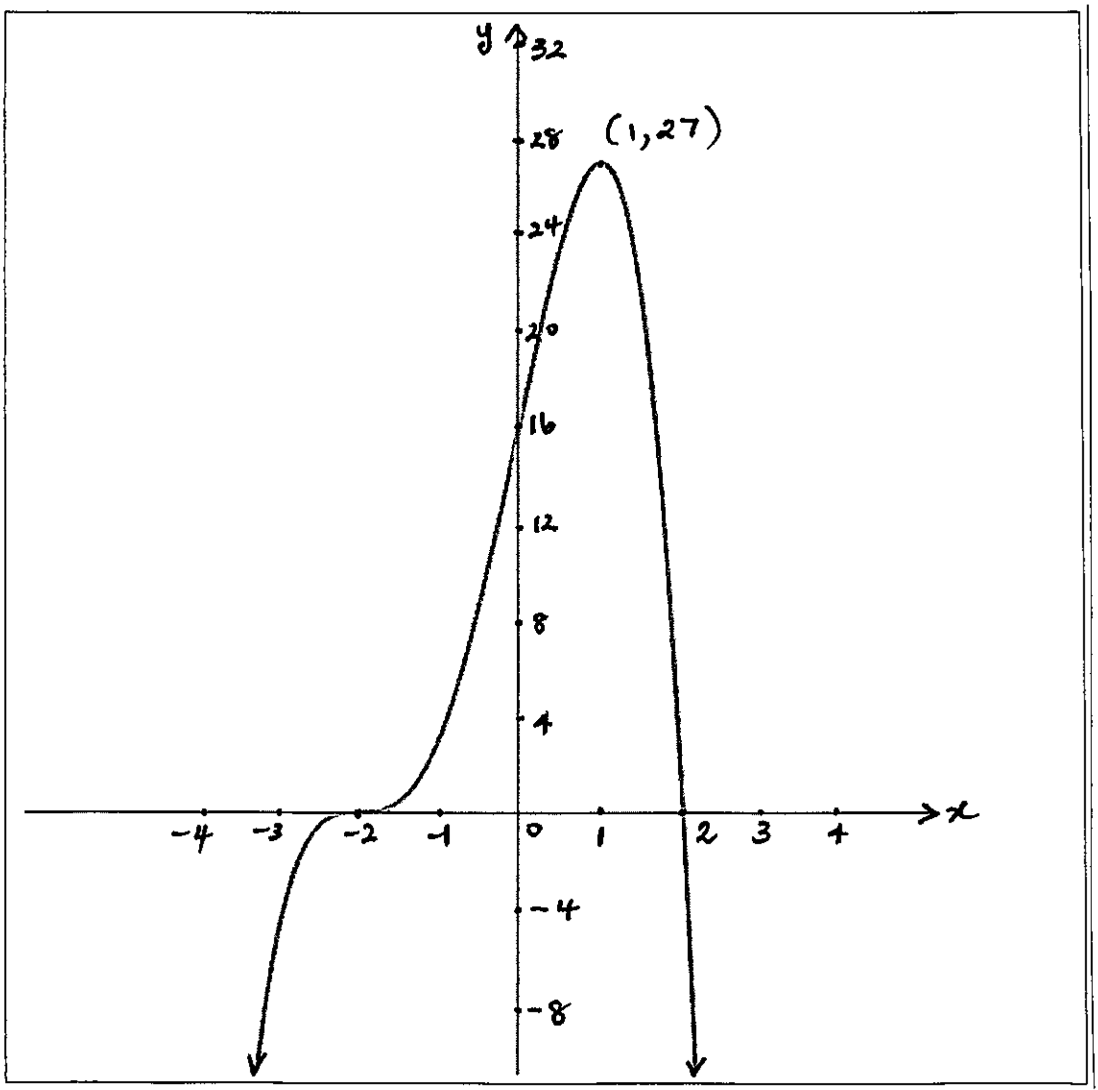
$y = 16$ $y = 0$ $\Big\}$ $(-2, 0)$ is a horizontal point of inflection from (b).

For $(0, 16)$:

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $f''(x)$ | 12 | 0 | $-36$ |
| Concavity | $\cup$ | $\cdot$ | $\cap$ |

$\therefore$ Since there is a change in concavity, $(0, 16)$ is a point of inflection.

(d) Sketch the graph of $y = f(x)$ below, showing all intercepts, stationary points, and points of inflection. **3**



# EXAMINER'S COMMENTS

Q20 (c) $\frac{1}{2}$ mark for both $x = 0$ & $x = -2$

$\frac{1}{2}$ mark for $f(0) = (2-0)(0+2)^3 = 2 \times 8 = 16$

OR $\frac{1}{2}$ mark for $y = 16$

- Some students showed both points of inflection in part (c), Marks transfered to part (b) accordingly.

$\frac{1}{2}$ mark for table with correct values

$\frac{1}{2}$ mark for stating since concavity changes, $(0,16)$ is a point of inflection.

Note: $(0,16)$ is not a horizontal point of inflection as $f'(0) \neq 0$, ie no stationary point at $x=0$.

Correct table for both points:

| $x$ | $-3$ | $-2$ | $-1$ | $0$ | $1$ |
|---|---|---|---|---|---|
| $f''(x)$ | $-36$ | $0$ | $12$ | $0$ | $-36$ |
| concavity | $\frown$ | $\cdot$ | $\smile$ | $\cdot$ | $\frown$ |

- Some students showed both points in part (b), marks transfered to part (c) accordingly.

(d) $\frac{1}{2}$ mk $\rightarrow$ $x$-intercept of $-2$

$\frac{1}{2}$ mk $\rightarrow$ $x$-intercept of $2$

$\frac{1}{2}$ mk $\rightarrow$ maximum turning point at $(1,27)$

$\frac{1}{2}$ mk $\rightarrow$ horizontal point of inflection at $(-2,0)$

$\frac{1}{2}$ mk $\rightarrow$ $y$-intercept at $16$

$\frac{1}{2}$ mk $\rightarrow$ smooth shape – both arrows pointing down, no horizontal point of inflection at $(0,16)$ $(-\frac{1}{2})$, no kink at $(2,0)$ $(-\frac{1}{2})$, curve should not flare out under $(2,0)$ $(-\frac{1}{2})$.

- Use a suitable scale on both axes with numbers on both.

(e) Does $y = f(x)$ have a global maximum or global minimum in its natural domain? If so, specify where. 1

$\underbrace{\text{Global maximum}}_{1/2}$ $\underbrace{\text{of 27 when } x = 1}_{1/2}$.

Note: There is no global minimum as the curve is continuous, and the lowest y-value cannot be determined in this domain.

**Question 21** (3 marks)

Find the exact value of $\int_3^4 \frac{x}{x^2-8}\,dx$ in simplest form. 3

$$= \frac{1}{2}\int_3^4 \frac{2x}{x^2-8}\,dx$$

$$= \frac{1}{2}\Big[\ln|x^2-8|\Big]_3^4$$

$$= \frac{1}{2}\Big[\ln|4^2-8| - \ln|3^2-8|\Big]$$

$$= \frac{1}{2}\Big[\ln|8| - \ln|1|\Big]$$

$$= \frac{1}{2}\Big[\ln 8 - 0\Big]$$

$$= \frac{1}{2}\ln 8$$

$$= \frac{1}{2}\ln 2^3$$

$$= \frac{3}{2}\ln 2$$

**End of Question 21**

**Proceed to Booklet 2 for Questions 22-29**

## EXAMINER'S COMMENTS

(e) $\frac{1}{2}$ mk for stating $y=f(x)$ has a global maximum

$\frac{1}{2}$ mk for specifying where.

Also accepted: $\underset{\frac{1}{2}}{\text{global maximum}}$ at $\underset{\frac{1}{2}}{(1, 27)}$

Q21

① mark for the $\frac{1}{2}$ outside the square brackets

① mark for $\ln|x^2-8|$ inside the square brackets

① mark for correct working leading to $\frac{1}{2}\ln 8$, OR $\frac{3}{2}\ln 2$ OR $\ln\sqrt{8}$ OR $\ln 2\sqrt{2}$, or equivalent.

· lost $\frac{1}{2}$ mark if you didn't include absolute value signs in your working.

Remember: $|8| = 8$

$|1| = 1$

$\ln 1 = 0$

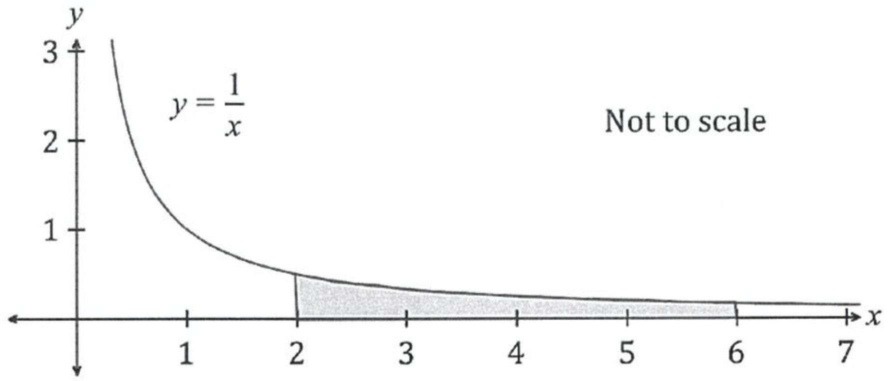
OR $\frac{1}{2}\left[\ln 8 - \ln 1\right]$

$= \frac{1}{2}\ln\left(\frac{8}{1}\right)$ (using log laws)

$= \frac{1}{2}\ln 8.$

**Question 22** (4 marks)

Consider the curve $y = \frac{1}{x}$ sketched below.




(a) Find the area bounded by the curve, the $x$-axis, and the lines $x = 2$ and $x = 6$ using the Trapezoidal Rule with five function values. Give your answer correct to three decimal places. **2**

$A \doteqdot \frac{b-a}{2n}\left\{f(a)+f(b)+2\left[f(x_1)+\dots+f(x_{n-1})\right]\right\}$

$\doteqdot \frac{6-2}{2\times 4}\left\{f(2)+f(6)+2\left[f(3)+f(4)+f(5)\right]\right\}$

$\doteqdot \frac{4}{8}\left\{\frac{1}{2}+\frac{1}{6}+2\left[\frac{1}{3}+\frac{1}{4}+\frac{1}{5}\right]\right\} \doteqdot 1.117 \text{ units}^2$

(b) Calculate the same area by evaluating $\int_2^6 \frac{1}{x}\,dx$. **1**

Give your answer correct to three decimal places.

$\int_2^6 \frac{1}{x}\,dx = \left[\ln|x|\right]_2^6$

$= \ln 6 - \ln 2$

$\doteqdot 1.099 \text{ units}^2$

**EXAMINER'S COMMENTS**

Common errors
- thinking there were 5 subintervals instead of 4
- wrong substituting when finding function values.

- 1 mark for applying the $\frac{b-a}{2n}$ part of formula.

- 1 mark for part inside brackets.

- $\frac{1}{2}$ mark lost if incurred a calculator error.

← 1 mark achieved at this point.
No half marks.

Most common error - not knowing how to integrate $\frac{1}{x}$.

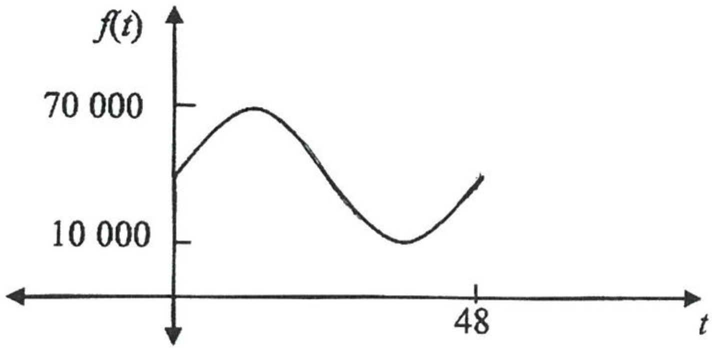
(c) Explain why there is a slight difference between your answers in part (a) and part (b). **1**

There is a slight difference as the trapezoidal rule (part a) gives an estimate or approximation, whereas integration (part b) gives an exact answer. Given the curve is concave up, the trapezoidal rule gives an overestimate, as the sides of trapezia lie above the curve.

**Question 23** (2 marks)

The function $f(t) = a\sin\left(\frac{\pi}{24}t\right) + b$ is drawn below, where $0 \le t \le 48$.

The maximum value of $f(t)$ is 70 000 when $t = 12$. The minimum value of $f(t)$ is 10 000 when $t = 36$.




What are the values of $a$ and $b$? **2**

$a$ = amplitude

$= \frac{70000 - 10000}{2}$

$= 30{,}000$

$b$ = centre

$= 10000 + 30000$

$= 40{,}000$

**EXAMINER'S COMMENTS**

← An answer similar to the first sentence is adequate to satisfy this particular question. Take note that a question phrased a bit differently may also require the information given in the second sentence.

1 mark for correct amplitude
1 mark for correct centre

**Question 24** (7 marks)

A particle starts to move from the origin along the $x$-axis.

Its velocity, measured in metres, at some time $t$ seconds, is given by $v = 4 - \frac{2}{3t+1}$ m/s.

(a) Explain why the particle is never at rest. **1**

If particle is at rest, $v = 0$

$4 - \frac{2}{3t+1} = 0$

$\frac{2}{3t+1} = 4$

$4(3t+1) = 2$

$12t + 4 = 2$

$12t = -2$

$t = -\frac{1}{6}$

$\therefore$ But $t \geq 0$

$\therefore$ The particle is never at rest.

(b) Find the acceleration of the particle at $t = 3$ seconds. **2**

$v = 4 - 2(3t+1)^{-1}$

$a = \frac{dv}{dt} = -2 \times -1(3t+1)^{-2} \times 3$

$a = \frac{6}{(3t+1)^2}$

When $t = 3$,

$a = \frac{6}{(3(3)+1)^2}$

$= \frac{6}{100}$

$= 0.06$ m/s$^2$

**EXAMINER'S COMMENTS**

← Accept an answer like this or similar that explains that $v \neq 0$ when $t > 0$.

- Some students showed that initial $v = 2$ m/s and then approaches $v = 4$ m/s, therefore never equalling zero.

1 mark for $a = \dfrac{6}{(3t+2)^2}$

1 mark for substitution of $t = 3$.
CFPE allowed if formula for a incorrect above.

Most Common error – forgetting to multiply by derivative of $(3t+1)$ when using chain rule.

(c) Find the exact distance travelled by the particle between $t = 0$ and $t = 5$ seconds. **3**

Method 1 (can be used as velocity doesn't change direction)

$$A = \int_0^5 4 - \frac{2}{3t+1}\,dt$$

$$= \int_0^5 4\,dt - \frac{2}{3}\int_0^5 \frac{3}{3t+1}\,dt$$

$$= \Big[4t\Big]_0^5 - \Big[\frac{2}{3}\ln|3t+1|\Big]_0^5$$

$$= 20 - 0 - \Big[\frac{2}{3}\ln|16| - \frac{2}{3}\ln|1|\Big]$$

$$= 20 - \frac{2}{3}\ln(16)$$

or equivalent.

Method 2

$$x = \int 4 - \frac{2}{3t+1}\,dt$$

$$x = \int 4\,dt - \frac{2}{3}\int \frac{3}{3t+1}\,dt$$

$$x = 4t - \frac{2}{3}\ln|3t+1| + c$$

SUB when $t=0$, $x=0$

$$0 = 4(0) - \frac{2}{3}\ln|1| + c$$

$$0 = 0 - 0 + c$$

$$\therefore c = 0$$

$$\therefore x = 4t - \frac{2}{3}\ln|3t+1|$$

when $t=5$,

$$x = 4(5) - \frac{2}{3}\ln|3(5)+1|$$

$$= 20 - \frac{2}{3}\ln(16)$$

(d) Find the particle's limiting velocity. **1**

As $t \to \infty$, $\frac{2}{3t+1} \to 0$

$$\therefore 4 - \frac{2}{3t+1} \to 4$$

$\therefore$ limiting velocity $= 4$ m/s

**EXAMINER'S COMMENTS**

← 1 mark for this line in either method.

← 1 mark for this line

← 1 mark for answer

← 1 mark for correct answer.

Many students did not understand the concept of limiting velocity being when $t \to \infty$.

**Question 25** (4 marks)

At the beginning of the year 1935, 100 cane toads were introduced into Australia. Exactly 5 years later, the population had grown to 1000.

Assume that the number of cane toads is increasing exponentially and satisfies an equation of the form

$$N = N_0 e^{kt}$$

where $N_0$ and $k$ are constants and $t$ is measured in years from the start of 1935.

(a) Show that $N_0 = 100$ and $k = \frac{\ln 10}{5}$. **2**

when $t=0$, $N=100$:

$100 = N_0 e^{k\times 0}$

$\therefore N_0 = 100$ 1

when $t=5$, $N=1000$:

$1000 = 100e^{5k}$

$10 = e^{5k}$

$\ln 10 = \cancel{\ln e}^{5k}$

$5k = \ln 10$

$\therefore k = \frac{\ln 10}{5}$ 1

(b) How long does it take for the population to reach 2 million? Give your answer to the nearest year. **2**

when $N = 2000\,000$

$2000\,000 = 100e^{\frac{\ln 10}{5}t}$ 1

$20000 = e^{\frac{\ln 10}{5}t}$

$\ln 20000 = \cancel{\ln e}^{\frac{\ln 10}{5}t}$

$\frac{\ln 10}{5}t = \ln 20000$

$t = \frac{\ln 20000}{\frac{\ln 10}{5}}$

$= 21.5051\ldots \approx 22$ years 1

**Question 26** (8 marks)

(a) Show that $\frac{d}{dx}(\ln(\sec x + \tan x)) = \sec x$. 3

$\text{LHS} = \frac{d}{dx}(\ln(\sec x + \tan x))$

$= \frac{1}{\sec x + \tan x} \times \frac{d}{dx}((\cos x)^{-1} + \tan x)$

$= \frac{1}{\sec x + \tan x} \times (-(\cos x)^{-2} \times -\sin x + \sec^2 x)$

$= \frac{1}{\sec x + \tan x} \times \left(\frac{\sin x \cdot 1}{\cos x \cdot \cos x} + \sec^2 x\right)$

$= \frac{1}{\sec x + \tan x} \times \left(\tan x \cdot \frac{1}{\cos x} + \sec^2 x\right)$

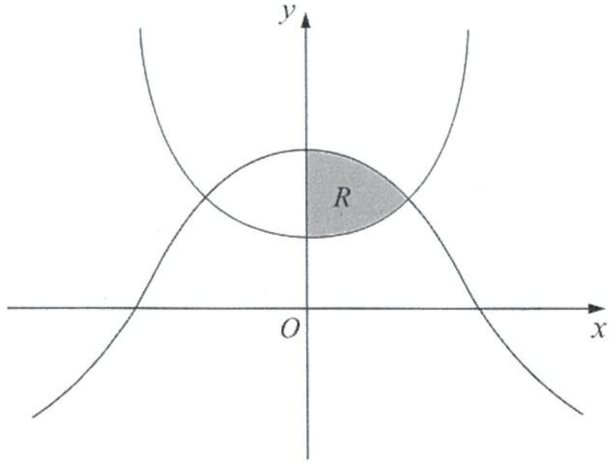
$= \frac{1}{\sec x + \tan x} \times (\tan x \sec x + \sec^2 x)$

$= \frac{\sec x(\cancel{\tan x + \sec x})}{\cancel{\sec x + \tan x}}$

$= \sec x$

$= \text{RHS}.$

The graph below shows the functions $y = \cos x$ and $y = \frac{1}{2}\sec x$.




(b) Show that the curves $y = \cos x$ and $y = \frac{1}{2}\sec x$ intersect in the first quadrant when $x = \frac{\pi}{4}$. **2**

$\cos x = \frac{1}{2}\sec x$

$2\cos x = \frac{1}{\cos x}$

$2\cos^2 x = 1$

$\cos^2 x = \frac{1}{2}$

$\therefore \cos x = \pm\frac{1}{\sqrt{2}}$ 1

$\therefore x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$ 1

$\therefore x = \frac{\pi}{4}$ in the first quadrant.

OR by substitution

(c) Hence, find the exact area of the shaded region $R$. **3**

$$R = \int_0^{\frac{\pi}{4}} \left(\cos x - \frac{1}{2}\sec x\right) dx$$

$$= \left[\sin x - \frac{1}{2}\ln(\sec x + \tan x)\right]_0^{\frac{\pi}{4}} \quad 1$$

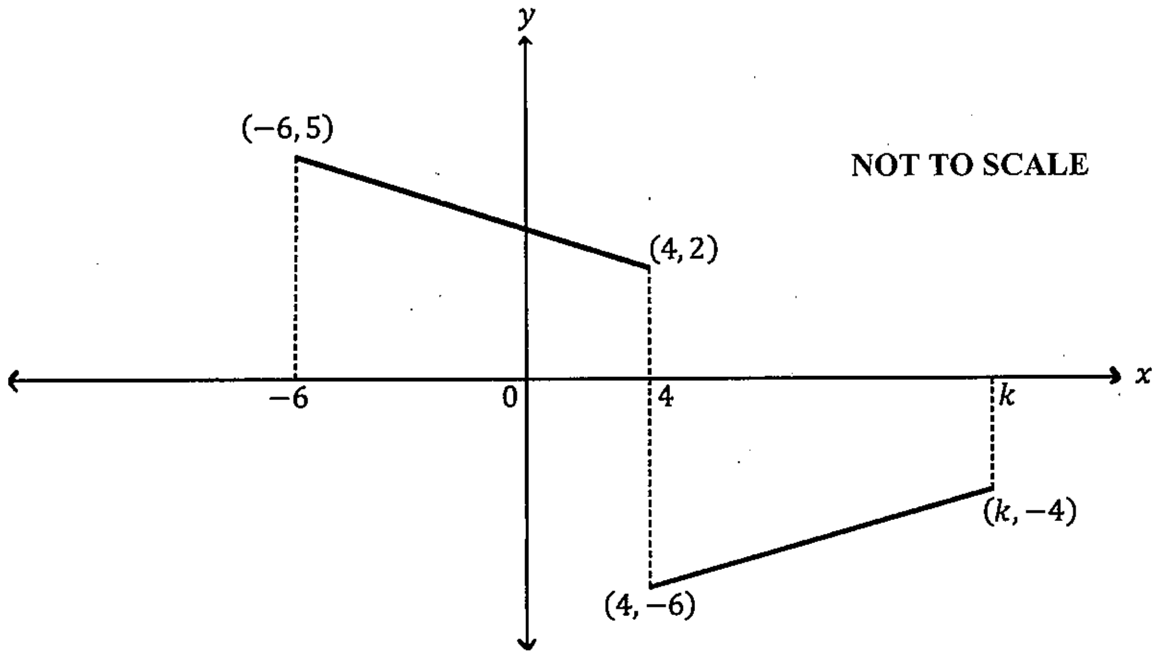
$$= \left(\sin\frac{\pi}{4} - \frac{1}{2}\ln\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right)\right) - \left(\sin 0 - \frac{1}{2}\ln(\sec 0 + \tan 0)\right) \quad 1$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{2}\ln(\sqrt{2}+1) - \left(0 - \frac{1}{2}\ln(1+0)\right)$$

$$= \frac{1}{\sqrt{2}} - \frac{1}{2}\ln(\sqrt{2}+1) \text{ units}^2 \quad 1$$

**Question 27** (3 marks)

Use the graph below to find the value of $k$ which satisfies $\int_{-6}^{k} f(x)\,dx = 0$. **3**




Area above $= \frac{1}{2}h(a+b)$

$= \frac{1}{2} \times 10 \times (5+2)$

$= 5 \times 7$

$= 35u^2$

Area below $= \frac{1}{2}h(a+b)$

$= \frac{1}{2} \times (k-4) \times (6+4)$

$= 5(k-4)$

$= 5k - 20 \ u^2$

For $\int_{-6}^{k} f(x)\,dx = 0$, area above = area below

$\therefore 5k - 20 = 35$

$5k = 55$

$\therefore k = 11$

(27)

## EXAMINER'S COMMENTS

Mostly well done, however many students did not recognise that the area under the curve was a trapezium.

The students who tried to find the equation of each line segment and then integrate struggled with the complicated and lengthy process.

Marking criteria:

1 – finding area above $x$-axis (or equivalent)

1 – finding area below $x$-axis (or equivalent)

1 – equating the areas, $k = 11$.

**Question 28** (4 marks)

Point $A$ $(2e, 1)$ lies on the function $h(x)$. The tangent to $h(x)$ at $A$ has equation $y = \frac{x}{2e}$.

Point $B$ is the image of the point $A$ on the function $g(x) = 3h(2x + 4)$.

(a) Show that $B$ has coordinates $(e - 2, 3)$. **1**

$g(x) = 3h(2(x+2))$

$\therefore$ from any point $(x, y)$ on $h(x)$, the image on $g(x)$ is $\left(\frac{x}{2} - 2, 3y\right)$

$\therefore B = \left(\frac{2e}{2} - 2, 1 \times 3\right)$

$= (e - 2, 3)$

(b) Hence, find the equation of the tangent to $g(x)$ at point $B$, in general form. **3**

$g(x) = 3h(2x+4)$

so $g'(x) = 3h'(2x+4) \times 2$

$= 6h'(2x+4)$

since $g'(e-2)$ is the gradient of the tangent at $B$,

then $g'(e-2) = 6h'(2(e-2)+4)$

$= 6h'(2e-4+4)$

$= 6h'(2e)$

$= 6 \times$ gradient of tangent at $A$.

$= 6 \times \frac{1}{2e}$

$= \frac{3}{e}$

$\therefore$ equation of tangent at $B$ is: $y - 3 = \frac{3}{e}(x - (e-2))$

$y - 3 = \frac{3}{e}(x - e + 2)$

$ey \cancel{-3e} = 3x \cancel{-3e} + 6$

$\therefore 3x - ey + 6 = 0$

28

## EXAMINER'S COMMENTS

(a) Mostly well done.

If kept function as $g(x) = 3h(2x+4)$, then the horizontal shift left 4 must be completed before the horizontal dilation of factor $\frac{1}{2}$.

(b) This part was poorly attempted by most students.

- $h(x)$ and $h(2x+4)$ is function notation — you cannot treat 'h' like a normal pronumeral.

- $y = \frac{x}{2e}$ is the tangent to $\underline{h(x)}$, not $g(x)$.

- general form means coefficient of $x$ is positive, and there should be no fractions in the equation.

MARKING CRITERIA

1 – for some correct progress to finding gradient of tangent to $g(x)$

1 – gradient of tangent to $g(x)$ at $B = \frac{3}{e}$.

1 – correct equation in general form.

(½ mark deducted if fractions in general form).

(28) **EXAMINER'S COMMENTS**

## ALTERNATIVE SOLUTIONS

(1) Tangent to $h(x)$ at $A$ is $y = \frac{1}{2e}x$

$\therefore$ gradient of $h(x)$ at $A$ is $\frac{1}{2e}$.

so gradient of $g(x)$ at $B$ is:

$m = \frac{1}{2e}$ vertically dilated by factor 3 and horizontally dilated by factor $\frac{1}{2}$.

$\therefore m_B = \frac{1 \times 3}{2e \times \frac{1}{2}}$

$= \frac{3}{e}$

$\vdots$

etc.

(2) Tangent to $g(x)$: $y = 3\left(\frac{2x+4}{2e}\right)$

$= 3\left(\frac{x+2}{e}\right)$

$= \frac{3x+2}{e}$

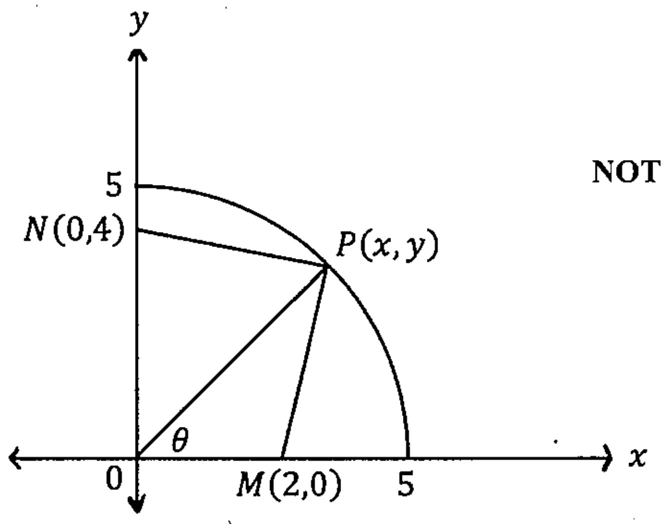
$= \frac{3x}{e} + \frac{2}{e}$

$\therefore$ gradient of tangent: $y' = \frac{3}{e}$

$\vdots$

etc

**Question 29** (7 marks)




NOT TO SCALE

The diagram above shows a part of the circle $x^2 + y^2 = 25$. The point $P(x, y)$ is on the circle, and point $O$ is the origin. Point $M$ has coordinates $(2, 0)$, point $N$ has coordinates $(0, 4)$, and $\angle MOP$ is measured in radians.

(a) Show that the area, $A$, of the quadrilateral $OMPN$ is given by **2**

$$A = 5\sin\theta + 10\cos\theta$$

$A_{\triangle OPM} = \frac{1}{2} \times OP \times OM \times \sin\theta$

$= \frac{1}{2} \times 5 \times 2 \times \sin\theta$

$= 5\sin\theta$

$A_{\triangle ONP} = \frac{1}{2} \times ON \times OP \times \sin\left(\frac{\pi}{2} - \theta\right)$

$= \frac{1}{2} \times 4 \times 5 \times \cos\theta$

$= 10\cos\theta$

$\therefore A_{OMPN} = A_{\triangle OPM} + A_{\triangle ONP}$

$= 5\sin\theta + 10\cos\theta$ as required.

29

## EXAMINER'S COMMENTS

(a) This part was mostly well done.

Remember that a show question requires all steps to be shown.

1 mark was awarded for area of $\Delta OPM$.
1 mark awarded for area of $\Delta ONP$.

(b) MARKING CRITERIA

1 - differentiating $A = 5\sin\theta + 10\cos\theta$ correctly.
2 - Solving for $\tan\theta = \frac{1}{2}$
3 - testing the stationary point and showing that it is a maximum.

($\frac{1}{2}$ mark was deducted if $\theta$ was in degrees)

(b) Find the value of $\tan\theta$ which gives the maximum area $A$. **3**

$A = 5\sin\theta + 10\cos\theta$

$\frac{dA}{d\theta} = 5\cos\theta - 10\sin\theta$

For max. area $\frac{dA}{d\theta} = 0$. $\therefore 5\cos\theta - 10\sin\theta = 0$

$10\sin\theta = 5\cos\theta$

$10\tan\theta = 5$

$\therefore \tan\theta = \frac{1}{2}$

$\theta = \tan^{-1}(\frac{1}{2})$

$= 0.463\ldots$ radians

Test $\theta = 0.463\ldots$: $\frac{d^2A}{d\theta^2} = -5\sin\theta - 10\cos\theta$

$= -5\sin(0.463\ldots) - 10\cos(0.463\ldots)$

$= 11.180\ldots < 0$

$\therefore$ concave down, maximum turning point

$\therefore$ Maximum area $A$ occurs when $\tan\theta = \frac{1}{2}$.

(c) Hence find, in surd form, the coordinates of point $P$ when $A$ is maximum. **2**

since $\tan\theta = \frac{1}{2} = m_{OP}$

$\therefore \frac{y}{x} = \frac{1}{2}$

$y = \frac{x}{2}$ — ①

Also $x^2 + y^2 = 25$ — ②

sub ① into ②: $x^2 + \left(\frac{x}{2}\right)^2 = 25$

$x^2 + \frac{x^2}{4} = 25$

$5x^2 = 100$

$x^2 = 20$

$\therefore x = \sqrt{20}$

$= 2\sqrt{5}$

when $x = 2\sqrt{5}$,

$y = \frac{2\sqrt{5}}{2}$

$= \sqrt{5}$

$\therefore P = (2\sqrt{5}, \sqrt{5})$

**END OF PAPER**

**EXAMINER'S COMMENTS**

(c)

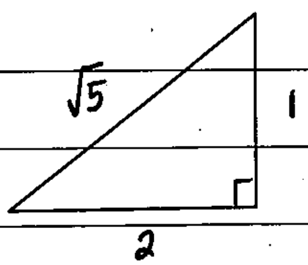
1 mark awarded for some correct progress

1 mark for final correct answer.

(1/2 mark deducted if not in surd form).

ALTERNATIVE SOLUTION

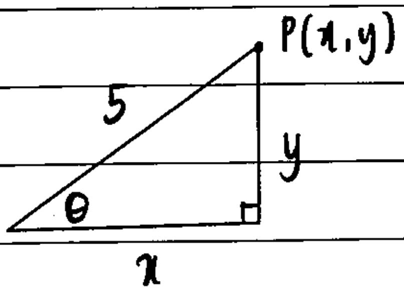
since $\tan\theta = \frac{1}{2}$:




$$h^2 = 1^2 + 2^2$$
$$\therefore h = \sqrt{1+4}$$
$$= \sqrt{5}$$

so $\sin\theta = \frac{1}{\sqrt{5}}$

and $\cos\theta = \frac{2}{\sqrt{5}}$




so $\sin\theta = \frac{y}{5}$

$$y = 5\sin\theta$$
$$= 5 \times \frac{1}{\sqrt{5}}$$
$$= \frac{5}{\sqrt{5}}$$
$$= \sqrt{5}$$

and $\cos\theta = \frac{x}{5}$

$$x = 5\cos\theta$$
$$= 5 \times \frac{2}{\sqrt{5}}$$
$$= \frac{10}{\sqrt{5}}$$
$$= 2\sqrt{5}$$

$$\therefore P(x,y) = (2\sqrt{5}, \sqrt{5})$$